Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

End of questions

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(8 marks)**

Let $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$, $\mathbf{b} = -4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{c} = -\mathbf{i} + 2\mathbf{j}$.

(a) Determine

(i) $\mathbf{b} - \mathbf{c}$. (1 mark)

Solution
$\begin{pmatrix}-4\\3\end{pmatrix} - \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix}$
Specific behaviours
✓ correct vector

(ii) $5\mathbf{c} + 3\mathbf{a}$. (2 marks)

Solution
$5\begin{pmatrix}-1\\2\end{pmatrix} + 3\begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}-5\\10\end{pmatrix} + \begin{pmatrix}9\\-15\end{pmatrix} = \begin{pmatrix}4\\-5\end{pmatrix}$
Specific behaviours
✓ determines scalar multiples ✓ correct vector

(iii) $|\mathbf{a} - \mathbf{c}|$. (2 marks)

Solution
$\begin{pmatrix}3\\-5\end{pmatrix} - \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}4\\-7\end{pmatrix}$ $\sqrt{4^2 + (-7)^2} = \sqrt{65}$
Specific behaviours
✓ determines sum ✓ correct value

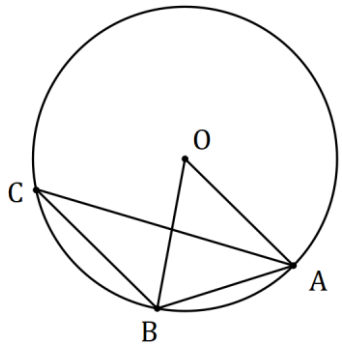
(b) Determine a unit vector that is parallel to $\mathbf{a} + \mathbf{c}$ but in the opposite direction. (3 marks)

Solution
$-\left(\begin{pmatrix}3\\-5\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}\right) = \begin{pmatrix}-2\\3\end{pmatrix}$ $\left\lvert\begin{pmatrix}-2\\3\end{pmatrix}\right\rvert = \sqrt{13}$ $\text{Soln: } \frac{1}{\sqrt{13}}\begin{pmatrix}-2\\3\end{pmatrix}$
Specific behaviours
✓ determines $-(\mathbf{a} + \mathbf{c})$ ✓ determines magnitude ✓ correct unit vector

See next page

Question 2 **(4 marks)**

In the diagram below (not drawn to scale) A, B and C lie on the circle with centre O and OA is parallel to CB.



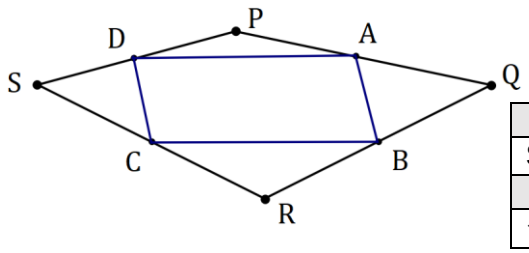
Determine, with reasons, the size of $\angle OBA$ and the size of $\angle ABC$ when $\angle OAC = 23°$.

Solution
$\angle ACB = \angle OAC = 23$ (Alternate angles) $\angle AOB = 2 \times \angle ACB = 46$ (Angle at centre) $\angle OBA = (180 - 46) \div 2 = 67°$ (Isosceles) $\angle OBC = \angle BOA = 46$ (Alternate angles) $\angle ABC = 46 + 67 = 113°$
Specific behaviours
✓ $\angle ACB$ with reason ✓ $\angle AOB$ with reason ✓ $\angle OBA$ with reason ✓ $\angle ABC$

End of questions

Question 3 **(9 marks)**

Quadrilateral $PQRS$ is shown below. The midpoints of sides PQ, QR, RS and SP are A, B, C and D respectively. Let $\overrightarrow{PQ} = 2\mathbf{q}, \overrightarrow{PR} = 2\mathbf{r}$ and $\overrightarrow{PS} = 2\mathbf{s}$.



Solution (a)
See diagram
Specific behaviours
✓ correct quadrilateral

(a) Sketch quadrilateral $ABCD$ on the diagram above. (1 mark)

(b) Determine expressions for $\overrightarrow{PB}, \overrightarrow{PC}$ and $\overrightarrow{CB}$ in terms of $\mathbf{q}, \mathbf{r}$ and $\mathbf{s}$. (4 marks)

Solution
$\overrightarrow{PB} = 2\mathbf{q} + \frac{1}{2}(2\mathbf{r} - 2\mathbf{q}) = \mathbf{q} + \mathbf{r}$ $\overrightarrow{PC} = 2\mathbf{s} + \frac{1}{2}(2\mathbf{r} - 2\mathbf{s}) = \mathbf{s} + \mathbf{r}$ $\overrightarrow{CB} = \overrightarrow{CP} + \overrightarrow{PB}$ $= (-\mathbf{s} - \mathbf{r}) + (\mathbf{q} + \mathbf{r}) = \mathbf{q} - \mathbf{s}$
Specific behaviours
✓ derives expression for $\overrightarrow{QR}$ or $\overrightarrow{SR}$ ✓ derives expression for $\overrightarrow{PB}$ ✓ derives expression for $\overrightarrow{PC}$ ✓ derives expression for $\overrightarrow{CB}$

(c) Prove that $\overrightarrow{DA} = \overrightarrow{CB}$ and $\overrightarrow{DC} = \overrightarrow{AB}$. (4 marks)

Solution
$\overrightarrow{DA} = \overrightarrow{PA} - \overrightarrow{PD}$ $= \mathbf{q} - \mathbf{s}$ $= \overrightarrow{CB}$ $\overrightarrow{DC} = \overrightarrow{PC} - \overrightarrow{PD}$ $= \mathbf{s} + \mathbf{r} - \mathbf{s}$ $= \mathbf{r}$ $\overrightarrow{AB} = \overrightarrow{PB} - \overrightarrow{PA}$ $= \mathbf{q} + \mathbf{r} - \mathbf{q}$ $= \mathbf{r}$ $= \overrightarrow{DC}$
Specific behaviours
✓ derives expression for $\overrightarrow{DA}$ ✓ equates $\overrightarrow{DA}$ to $\overrightarrow{CB}$ ✓ derives expression for $\overrightarrow{AB}$ ✓ derives expression for $\overrightarrow{DC}$ and equates to $\overrightarrow{AB}$

See next page

Question 4 **(6 marks)**

(a) Body A moves with a velocity of $6\sqrt{2}\mathbf{i} - 6\sqrt{2}\mathbf{j}$ ms^{-1}. Determine the speed of this body and the bearing it is travelling in. (3 marks)

Solution
$s^2 = \left(6\sqrt{2}\right)^2 + \left(-6\sqrt{2}\right)^2$ $s = 12 \text{ m/s}$ $\angle(x\text{-axis}) = -45°$ $\text{Bearing} = 90 + 45 = 135°$
Specific behaviours
✓ correct speed ✓ angle with x-axis ✓ correct bearing

(b) Body B moves 32 m on a bearing of $300°$. Express this displacement in component form using unit vectors **i** and **j**. (3 marks)

Solution
$\angle(x\text{-axis}) = 150°$ $\mathbf{r} = 32\cos(150°)\,\mathbf{i} + 32\sin(150°)\,\mathbf{j}$ $= -16\sqrt{3}\mathbf{i} + 16\mathbf{j}$
Specific behaviours
✓ correct angle from x-axis ✓ correct **i**-coefficient ✓ correct **j**-coefficient

End of questions

Question 5 **(7 marks)**

(a) The work done, in joules, by a force of $\mathbf{F}$ Newtons in change in the displacement of an object by $\mathbf{s}$ metres, is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$. Determine the work done by

(i) force $\mathbf{F} = (5\mathbf{i} + 10\mathbf{j})$ N that moves a small body from $(16\mathbf{i} - 2\mathbf{j})$ m to $(22\mathbf{i} + 8\mathbf{j})$ m. (2 marks)

Solution
$\begin{pmatrix}22\\8\end{pmatrix} - \begin{pmatrix}16\\-2\end{pmatrix} = \begin{pmatrix}6\\10\end{pmatrix}$ $w = \begin{pmatrix}5\\10\end{pmatrix} \cdot \begin{pmatrix}6\\10\end{pmatrix} = 30 + 100 = 130 \text{ J}$
Specific behaviours
✓ displacement vector ✓ correct work done

(ii) a horizontal force of 45 N that pushes a small body 0.4 m up a slope inclined at $45°$ to the horizontal. (2 marks)

Solution
$w = 45 \times 0.4 \times \cos 45$ $= 45 \times 0.4 \times \frac{\sqrt{2}}{2}$ $= 9\sqrt{2} \text{ J}$
Specific behaviours
✓ uses correct expression ✓ correct work done

(b) Determine the vector projection of $(-\mathbf{i} - 4.5\mathbf{j})$ on $(3\mathbf{i} - 4\mathbf{j})$. (3 marks)

Solution
$\begin{pmatrix}-1\\-4.5\end{pmatrix} \cdot \begin{pmatrix}3\\-4\end{pmatrix} = 15$ $\begin{pmatrix}3\\-4\end{pmatrix} \cdot \begin{pmatrix}3\\-4\end{pmatrix} = 25$ $\frac{15}{25}\begin{pmatrix}3\\-4\end{pmatrix} = \begin{pmatrix}9/5\\-12/5\end{pmatrix}$
Specific behaviours
✓ scalar products ✓ substitutes into expression ✓ correct vector projection

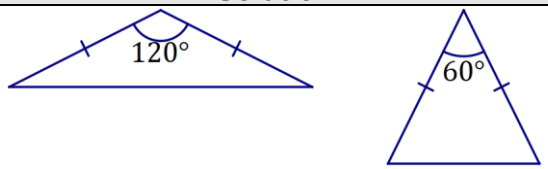
See next page

Question 6 **(6 marks)**

Consider the following statement that refers to two **isosceles** triangles.

If the triangles have the same area, then the triangles are congruent.

(a) Use a counter-example to explain why the statement is false. (2 marks)

Solution
 The isosceles triangles shown have the same area but are not congruent.
Specific behaviours
✓ sketches two isosceles triangles ✓ shows supplementary angles and congruent sides Or ✓ shows dimensions that give same area

(b) Write the converse statement and state whether it is true or false. (2 marks)

Solution
If the triangles are congruent, then the triangles have the same area. This statement is true.
Specific behaviours
✓ correct inverse statement ✓ states true

(c) Write the contrapositive statement and state whether it is true or false. (2 marks)

Solution
If the triangles are not congruent, then the triangles do not have the same area. This statement is false.
Specific behaviours
✓ correct contrapositive statement ✓ states false

End of questions

Question 7 **(8 marks)**

(a) Evaluate $\frac{^{55}P_2}{^{6}P_3}$. (3 marks)

Solution
$\frac{^{55}P_2}{^{6}P_3} = \frac{55!}{53!} \div \frac{6!}{3!}$ $= \frac{55 \times 54}{6 \times 5 \times 4}$ $= \frac{99}{4}$
Specific behaviours
✓ expresses as factorials ✓ eliminates factorials ✓ correct value, simplified

(b) (i) Given that $^{n}P_{r+1} = k \times {}^{n}P_r$, determine the constant k in terms of n and/or r. (3 marks)

Solution
$^{n}P_{r+1} = \frac{n!}{(n-r-1)!}$ $= \frac{(n-r)n!}{(n-r)(n-r-1)!}$ $= \frac{(n-r)n!}{(n-r)!}$ $= (n-r) \times {}^{n}P_r$ $\therefore k = n - r$
Specific behaviours
✓ expresses LHS using factorials ✓ factors out term from denominator ✓ correct expression

(ii) Given that $^{13}P_8 = 51\,891\,840$, determine $^{13}P_{10}$. (2 marks)

Solution
$^{13}P_{10} = 4 \times {}^{13}P_9 = 4 \times 5 \times {}^{13}P_8 = 20 \times {}^{13}P_8$ $20 \times 51891840 = 1\,037\,836\,800$
Specific behaviours
✓ correct multiplier ✓ correct value

See next page

Question 8 **(4 marks)**

Consider the statement

If $m > m^2$, with $m \in \mathbb{R}$, then $m < 1$.

(a) State the converse of this statement. (1 mark)

Solution
If $m < 1$, with $m \in \mathbb{R}$, then $m > m^2$.
Specific behaviours
✓ correct statement (comment but no deduction if missing $m \in \mathbb{R}$)

(b) Is the converse always true? If yes, then explain why it is always true; if not, provide a counter example and adjust the domain of m so that the converse is always true.

(3 marks)

Solution
It is not always true because it does not work for $m \leq 0$. Counter example: e.g. if $m = -2 < 1$ then $m^2 = 4 > m$ i.e. False Adjusted domain: Converse is always true for $0 < m < 1$
Specific behaviours
✓ states that the converse is not always true ✓ provides a specific counterexample ✓ correct adjusted domain

End of questions

Supplementary page

Question number: _____